NONHELICAL INVERSE TRANSFER OF A DECAYING TURBULENT MAGNETIC FIELD

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Abstract—In the presence of magnetic helicity, inverse transfer from small to large scales is well known in magnetohydrodynamic (MHD) turbulence and has applications in astrophysics, cosmology, and fusion plasmas. Using high resolution direct numerical simulations of magnetohydrodynamically dominated small scales, we show in Fig. 2 representations of the spectral transfer function in incompressible [7] and relativistic [8] simulations, where this idea may also apply to incompressible [7] and relativistic [8] simulations, where inverse nonhelical transfer has recently been confirmed.

DECAY SIMULATIONS

We solve the compressible MHD equations for u, the gas density ρ at constant sound speed cs, and the magnetic vector potential A, so that B = ∇ × A. Following our earlier work [1, 2, 3], we initialize our decaying DNS by restarting them from a snapshot of a driven DNS, where a random forcing was applied in the evolution equation for A rather than u. To allow for sufficient scale separation, we take k0/k1 = 60. We use the PENCIL Code (http://pencil-code.googlecode.com/) at a resolution of 23043 meshpoints on 9216 processors. The code uses sixth order finite differences and a third order accurate time stepping scheme. Our magnetic and kinetic energy spectra are normalized such that ∫ Eₘ(k, t) dk = Eₘ(t) = v₂/2 and ∫ Eᵥ(k, t) dk = Eᵥ(t) = v₂rms/2 are magnetic and kinetic energies per unit mass. The magnetic integral scale is defined as kₘ = kₘ₋₁(t) = ∫ k⁻¹ Eₘ(k, t) dk/Eₘ(t). Time is given in initial Alfvén times τₐ = (υ₀k₀)^⁻¹.

In Fig. 1 we show Eₘ(k, t) and Eᵥ(k, t). We find an inertial range with weak turbulence scaling,

EₘWT(k, t) = CₘWT(v₂k₀k₃)¹/₂k⁻², (1)

where k⁻¹ₘ(t) = ∫ k⁻¹ Eₘ(k, t) dk/Eₘ(t) is the integral scale and kₙ has been used in place of kₐ. The prefactor is CₘWT ≈ 1.9; see the inset. In agreement with earlier work [2, 4], Eₘ decays like t⁻¹.

At small wavenumbers the k⁴ and k⁻² subinertial ranges respectively for Eₘ(k, t) and Eᵥ(k, t) are carried over from the initial conditions. The k⁴ Batchelor spectrum is in agreement with the causality requirement [5, 6] for the divergence-free vector field B. The velocity is driven entirely by the magnetic field and follows a white noise spectrum, Eᵥ(k) ∝ k² [6]. The resulting difference in the scaling implies that, although magnetic energy dominates over kinetic, the two spectra must cross at sufficiently small wavenumbers. This idea may also apply to incompressible [7] and relativistic [8] simulations, where inverse nonhelical transfer has recently been confirmed.

NATURE OF INVERSE TRANSFER

To quantify the nature of inverse transfer we show in Fig. 2 representations of the spectral transfer function Tᵥₚg = ⟨ Ĵₗk · (ūₚ × B̄g) ⟩ and compare with the corresponding helical case of Ref. [3], but with 1024³ mesh points and at a comparable time. Here, the superscripts indicate the radius of a shell in wavenumber space of Fourier filtered vector
fields; see Ref. [9] for such an analysis in driven helical turbulence. The transfer function $T_{kpq}$ quantifies the gain of magnetic energy at wavenumber $k$ from interactions of velocities at wavenumber $p$ and magnetic fields at wavenumber $q$. Fig. 2(a) shows a gain for $k/k_0 < 0.1$, which is about half of that for the helical case. The corresponding losses for $k/k_0 > 0.1$ are about equal in the two cases. In both cases, the magnetic gain at $k/k_0 = 0.07 = 4/60$ results from $u^p$ with $0 < p/k_0 < 0.2$ interacting with $B^q$ at $q/k_0 > 0.1$; see the light yellow shades in Fig. 2(b). Note that work done by the Lorentz force is $(u^p \cdot (J^k \times B^q)) = -T_{kpq}$. Thus, negative values of $T_{kpq}$ quantify the gain of kinetic energy at wavenumber $p$ from interactions of magnetic fields at wavenumbers $k$ and $q$. Blue dark shades in Fig. 2(c) indicate therefore that the gain of kinetic energy at $p/k_0 = 0.07$ results from magnetic interactions at wavenumbers $k$ and $q$ of around $0.1k_0$. These results support the interpretation that the increase of spectral power at large scales is similar to the inverse transfer in the helical case; see [10] for information concerning the total energy transfer.

To exclude that the inverse energy transfer is a consequence of the invariance of magnetic helicity, $\mathcal{H}_M(t) = (A \cdot B)$, we compare $\xi_M$ with its lower bound $\xi_{M_{\text{min}}} = |\mathcal{H}_M|/2E_M$ [2]. In nonhelical MHD turbulence, $\xi_M$ is known to grow like $t^{1/2}$ [2, 4]. Even though the initial condition was produced with nonhelical plane waves, we find $\mathcal{H}_M \neq 0$ due to fluctuations. Since $\mathcal{H}_M$ is conserved and $\xi_{M_{\text{min}}}$ decays like $t^{-1}$, $\xi_{M_{\text{min}}}$ grows linearly and faster than $\xi_M \sim t^{1/2}$, so they will meet at $t/\tau_A = 10^5$ and then continue to grow as $t^{-2/3}$, but at $t/\tau_A = 10^5$ this cannot explain the inverse transfer. By contrast, we cannot exclude the possibility of the quasi two-dimensional mean squared vector potential, $\langle A_{2D}^2 \rangle$, being approximately conserved [10]. This could explain the $\xi_M \sim t^{1/2}$ scaling and the inverse transfer if the flow was locally two-dimensional [11].

Our results support the idea of the weak turbulence $k^{-2}$ scaling for strong magnetic field that is here for the first time globally isotropic and not an imposed one [13]. At small scales, however, approximate equipartition is still possible. The decay is slower than for usual MHD turbulence which is arguably governed by the Liozynskiy invariant [14]. Future investigations of the differences between these types of turbulence are warranted [10]. Interestingly, the extended plateau in the velocity spectrum around the position of the magnetic peak may be important for producing observationally detectable broad gravitational wave spectra [15].

References